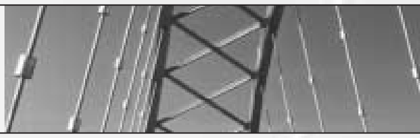


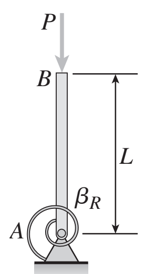
11

Columns

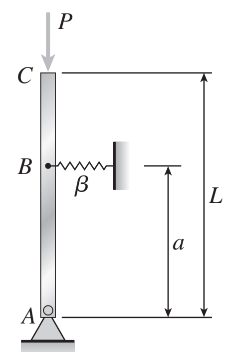
Idealized Buckling Models

Problem 11.2-1 through 11.2-4 The figure shows an idealized structure consisting of one or more **rigid bars** with pinned connections and linearly elastic springs. Rotational stiffness is denoted β_R and translational stiffness is denoted β.

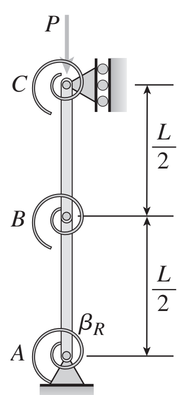
Determine the critical load P_{cr} for the structure.



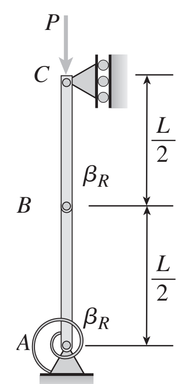
Prob. 11.2-1



Prob. 11.2-2

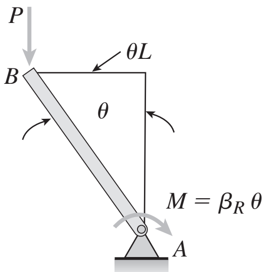


Prob. 11.2-3



Prob. 11.2-4

Solution 11.2-1 **Rigid bar *AB***

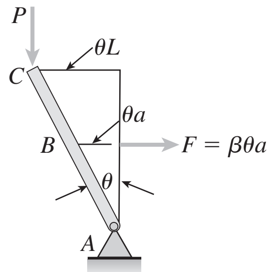


$$\sum M_A = 0$$

$$P(\theta L) - \beta_R \theta = 0$$

$$P_{cr} = \frac{\beta_R}{L} \quad \longleftarrow$$

Solution 11.2-2 **Rigid bar *ABC***

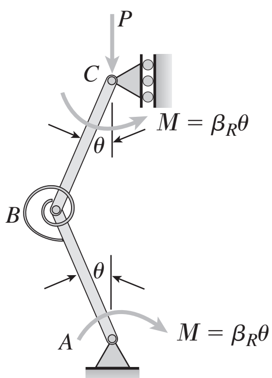


$$\sum M_A = 0$$

$$P\theta L - \beta\theta a^2 = 0$$

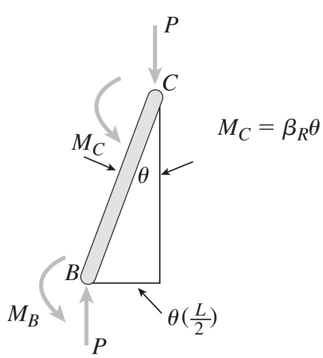
$$P_{cr} = \frac{\beta a^2}{L} \quad \longleftarrow$$

Solution 11.2-3 **Two rigid bars with a pin connection**



$\sum M_A = 0$ Shows that there are no horizontal reactions at the supports.

FREE-BODY DIAGRAM OF BAR *BC*



$$M_C = \beta_R\theta$$

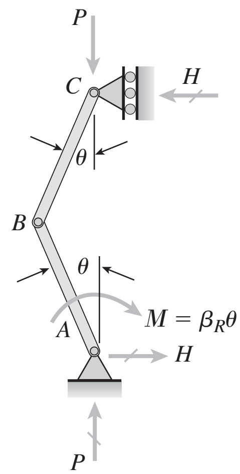
$$M_B = \beta_R(2\theta)$$

$$\sum M_B = 0 \quad M_B + M_C - P\theta\left(\frac{L}{2}\right) = 0$$

$$\beta_R(2\theta) + \beta_R\theta = \frac{PL\theta}{2}$$

$$P_{cr} = \frac{6\beta_R}{L} \quad \longleftarrow$$

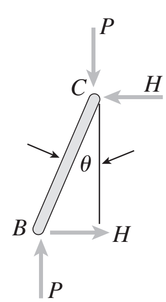
Solution 11.2-4 **Two rigid bars with a pin connection**



$$\sum M_A = 0 \quad HL - \beta_R\theta = 0$$

$$H = \frac{\beta_R\theta}{L}$$

FREE-BODY DIAGRAM OF BAR *BC*



$$\sum M_B = 0 \quad H\left(\frac{L}{2}\right) - P\left(\frac{\theta L}{2}\right) = 0$$

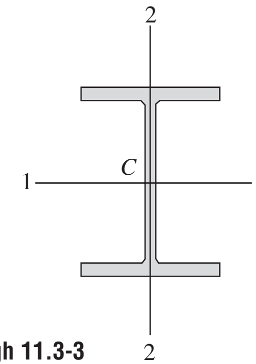
$$P_{cr} = \frac{H}{\theta} = \frac{\beta_R}{L} \quad \longleftarrow$$

Critical Loads of Columns with Pinned Supports

The problems for Section 11.3 are to be solved using the assumptions of ideal, slender, prismatic, linearly elastic columns (Euler buckling). Buckling occurs in the plane of the figure unless stated otherwise.

Problem 11.3-1 Calculate the critical load P_{cr} for a W 8 × 35 steel column (see figure) having length $L = 24$ ft and $E = 30 \times 10^6$ psi under the following conditions:

(a) The column buckles by bending about its strong axis (axis 1-1), and (b) the column buckles by bending about its weak axis (axis 2-2). In both cases, assume that the column has pinned ends.



Probs. 11.3-1 through 11.3-3

Solution 11.3-1 Column with pinned supports

W 8 × 35 steel column

$L = 24 \text{ ft} = 288 \text{ in.} \quad E = 30 \times 10^6 \text{ psi}$

$I_1 = 127 \text{ in.}^4 \quad I_2 = 42.6 \text{ in.}^4 \quad A = 10.3 \text{ in.}^2$

(a) Buckling about strong axis

$$P_{cr} = \frac{\pi^2 EI_1}{L^2} = 453 \text{ k} \longleftarrow$$

(b) Buckling about weak axis

$$P_{cr} = \frac{\pi^2 EI_2}{L^2} = 152 \text{ k} \longleftarrow$$

Note: $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{453 \text{ k}}{10.3 \text{ in.}^2} = 44 \text{ ksi}$

$\therefore$ Solution is satisfactory if $\sigma_{PL} \geq 44$ ksi

Problem 11.3-2 Solve the preceding problem for a W 10 × 60 steel column having length $L = 30$ ft.

Solution 11.3-2 Column with pinned supports

W 10 × 60 steel column

$L = 30 \text{ ft} = 360 \text{ in.} \quad E = 30 \times 10^6 \text{ psi}$

$I_1 = 341 \text{ in.}^4 \quad I_2 = 116 \text{ in.}^4 \quad A = 17.6 \text{ in.}^2$

(a) Buckling about strong axis

$$P_{cr} = \frac{\pi^2 EI_1}{L^2} = 779 \text{ k} \longleftarrow$$

(b) Buckling about weak axis

$$P_{cr} = \frac{\pi^2 EI_2}{L^2} = 265 \text{ k} \longleftarrow$$

Note: $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{779 \text{ k}}{17.6 \text{ in.}^2} = 44 \text{ ksi}$

$\therefore$ Solution is satisfactory if $\sigma_{PL} \geq 44$ ksi

Problem 11.3-3 Solve Problem 11.3-1 for a W 10 × 45 steel column having length $L = 28$ ft.

Solution 11.3-3 Column with pinned supports

W 10 × 45 steel column

$L = 28 \text{ ft} = 336 \text{ in.} \quad E = 30 \times 10^6 \text{ psi}$

$I_1 = 248 \text{ in.}^4 \quad I_2 = 53.4 \text{ in.}^4 \quad A = 13.3 \text{ in.}^2$

(a) Buckling about strong axis

$$P_{cr} = \frac{\pi^2 EI_1}{L^2} = 650 \text{ k} \longleftarrow$$

(b) Buckling about weak axis

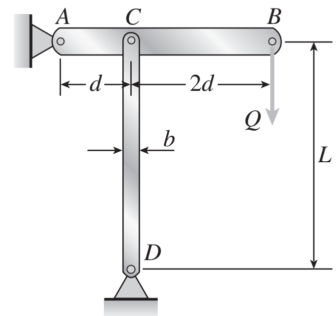
$$P_{cr} = \frac{\pi^2 EI_2}{L^2} = 140 \text{ k} \longleftarrow$$

Note: $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{650 \text{ k}}{13.3 \text{ in.}^2} = 49 \text{ ksi}$

$\therefore$ Solution is satisfactory if $\sigma_{PL} \geq 49$ ksi

Problem 11.3-4 A horizontal beam AB is pin-supported at end A and carries a load Q at end B, as shown in the figure. The beam is supported at C by a pinned-end column. The column is a solid steel bar ($E = 200$ GPa) of square cross section having length $L = 1.8$ m and side dimensions $b = 60$ mm.

Based upon the critical load of the column, determine the allowable load Q if the factor of safety with respect to buckling is $n = 2.0$.



Probs. 11.3-4 and 11.3-5

Solution 11.3-4 Beam supported by a column

COLUMN CD (STEEL)

$E = 200$ GPa $\quad L = 1.8$ m

Square cross section: $b = 60$ mm

Factor of safety: $n = 2.0$

$$I = \frac{b^4}{12} = 1.08 \times 10^6 \text{ mm}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = 657.97 \text{ kN}$$

BEAM ACB $\quad \sum M_A = 0 \quad Q = \frac{P}{3}$

$$Q_{allow} = \frac{P_{allow}}{3} = \frac{P_{cr}}{3n} = \frac{P_{cr}}{6.0} = 109.7 \text{ kN} \quad \longleftarrow$$

Problem 11.3-5 Solve the preceding problem if the column is aluminum ($E = 10 \times 10^6$ psi), the length $L = 30$ in., the side dimension $b = 1.5$ in., and the factor of safety $n = 1.8$.

Solution 11.3-5 Beam supported by a column

COLUMN CD (STEEL)

$E = 10 \times 10^6$ psi $\quad L = 30$ in.

Square cross section: $b = 1.5$ in.

Factor of safety: $n = 1.8$

$$I = \frac{b^4}{12} = 0.42188 \text{ in.}^4$$

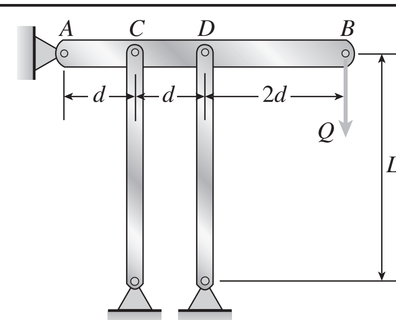
$$P_{cr} = \frac{\pi^2 EI}{L^2} = 46.264 \text{ k}$$

BEAM ACB $\quad \sum M_A = 0 \quad Q = \frac{P}{3}$

$$Q_{allow} = \frac{P_{allow}}{3} = \frac{P_{cr}}{3n} = \frac{P_{cr}}{5.4} = 8.57 \text{ k} \quad \longleftarrow$$

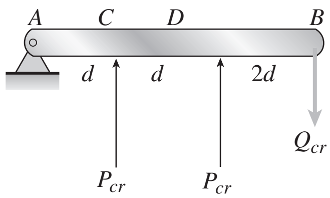
Problem 11.3-6 A horizontal beam AB is pin-supported at end A and carries a load Q at end B, as shown in the figure. The beam is supported at C and D by two identical pinned-end columns of length L. Each column has flexural rigidity EI.

What is the critical load Q_{cr}? (In other words, at what load Q_{cr} does the system collapse because of Euler buckling of the columns?)



Solution 11.3-6 **Beam supported by two columns**

Collapse occurs when both columns reach the critical load.

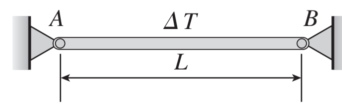


$$\sum M_A = 0 \qquad Q_{cr} = \frac{3P_{cr}}{4}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad \therefore\ Q_{cr} = \frac{3\pi^2 EI}{4L^2} \leftarrow$$

Problem 11.3-7 A slender bar AB with pinned ends and length L is held between immovable supports (see figure).

What increase ΔT in the temperature of the bar will produce buckling at the Euler load?



Solution 11.3-7 **Bar with immovable pin supports**

L = length $\quad A$ = cross-sectional area

I = moment of inertia $\quad E$ = modulus of elasticity

α = coefficient of thermal expansion

ΔT = uniform increase in temperature

AXIAL COMPRESSIVE FORCE IN BAR (EQ. 2-17)

$$P = EA\alpha(\Delta T)$$

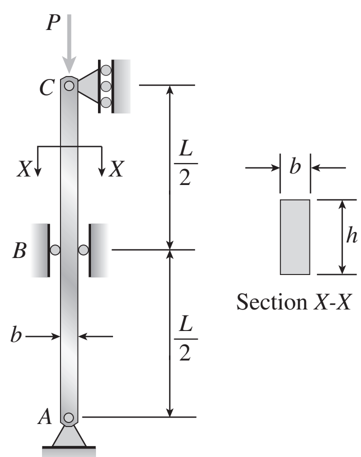
EULER LOAD $\quad P_{cr} = \dfrac{\pi^2 EI}{L^2}$

INCREASE IN TEMPERATURE TO PRODUCE BUCKLING

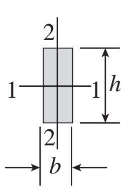
$$P = P_{cr} \qquad EA\alpha(\Delta T) = \frac{\pi^2 EI}{L^2} \qquad \Delta T = \frac{\pi^2 I}{\alpha A L^2} \leftarrow$$

Problem 11.3-8 A rectangular column with cross-sectional dimensions b and h is pin-supported at ends A and C (see figure). At midheight, the column is restrained in the plane of the figure but is free to deflect perpendicular to the plane of the figure.

Determine the ratio h/b such that the critical load is the same for buckling in the two principal planes of the column.



Solution 11.3-8 **Column with restraint at midheight**



Critical loads for buckling about axes 1-1 and 2-2:

$$P_1 = \frac{\pi^2 EI_1}{L^2} \qquad P_2 = \frac{\pi^2 EI_2}{(L/2)^2} = \frac{4\pi^2 EI_2}{L^2}$$

FOR EQUAL CRITICAL LOADS

$$P_1 = P_2 \qquad \therefore\ I_1 = 4I_2$$

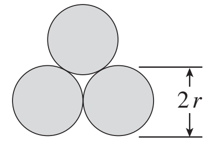
$$I_1 = \frac{bh^3}{12} \qquad I_2 = \frac{hb^3}{12}$$

$$bh^3 = 4hb^3 \qquad \frac{h}{b} = 2 \leftarrow$$

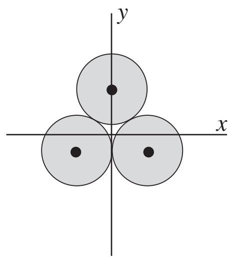
Problem 11.3-9 Three identical, solid circular rods, each of radius r and length L, are placed together to form a compression member (see the cross section shown in the figure).

Assuming pinned-end conditions, determine the critical load P_{cr} as follows: (a) The rods act independently as individual columns, and (b) the rods are bonded by epoxy throughout their lengths so that they function as a single member.

What is the effect on the critical load when the rods act as a single member?



Solution 11.3-9 **Three solid circular rods**



R = Radius L = Length

(a) Rods act independently

$$P_{cr} = \frac{\pi^2 EI}{L^2}(3) \qquad I = \frac{\pi r^4}{4}$$

$$P_{cr} = \frac{3\pi^3 E r^4}{4L^2} \quad \longleftarrow$$

(b) Rods are bonded together

The x and y axes have their origin at the centroid of the cross section. Because there are three different centroidal axes of symmetry, all centroidal axes are principal axes and all centroidal moments of inertia are equal (see Section 12.9).

From Case 9, Appendix D:

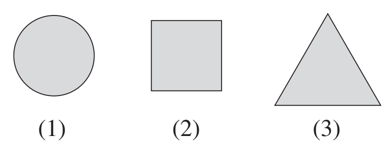
$$I = I_Y = \frac{\pi r^4}{4} + 2\left(\frac{5\pi r^4}{4}\right) = \frac{11\pi r^4}{4}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{11\pi^3 E r^4}{4L^2} \quad \longleftarrow$$

Note: Joining the rods so that they act as a single member increases the critical load by a factor of 11/3, or 3.67. ⟵

Problem 11.3-10 Three pinned-end columns of the same material have the same length and the same cross-sectional area (see figure). The columns are free to buckle in any direction. The columns have cross sections as follows: (1) a circle, (2) a square, and (3) an equilateral triangle.

Determine the ratios $P_1 : P_2 : P_3$ of the critical loads for these columns.



Solution 11.3-10 **Three pinned-end columns**

E, L, and A are the same for all three columns.

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad \therefore P_1 : P_2 : P_3 = I_1 : I_2 : I_3$$

(1) Circle Case 9, Appendix D

$$I = \frac{\pi d^4}{64} \qquad A = \frac{\pi d^2}{4} \qquad \therefore I_1 = \frac{A^2}{4\pi}$$

(2) Square Case 1, Appendix D

$$I = \frac{b^4}{12} \qquad A = b^2 \qquad \therefore I_2 = \frac{A^2}{12}$$

(3) Equilateral triangle Case 5, Appendix D

$$I = \frac{b^4\sqrt{3}}{96} \qquad A = \frac{b^2\sqrt{3}}{4} \qquad \therefore I_3 = \frac{A^2\sqrt{3}}{18}$$

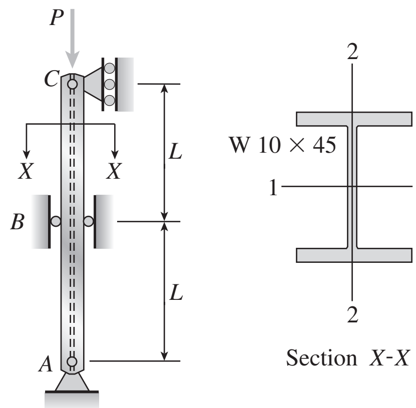
$$P_1 : P_2 : P_3 = I_1 : I_2 : I_3 = 1 : \frac{\pi}{3} : \frac{2\pi\sqrt{3}}{9}$$

$$= 1.000 : 1.047 : 1.209 \quad \longleftarrow$$

Note: For each of the above cross sections, every centroidal axis has the same moment of inertia (see Section 12.9).

Problem 11.3-11 A long slender column *ABC* is pinned at ends *A* and *C* and compressed by an axial force *P* (see figure). At the midpoint *B*, lateral support is provided to prevent deflection in the plane of the figure. The column is a steel wide-flange section (W 10 × 45) with $E = 30 \times 10^6$ psi. The distance between lateral supports is $L = 18$ ft.

Calculate the allowable load *P* using a factor of safety $n = 2.4$, taking into account the possibility of Euler buckling about either principal centroidal axis (i.e., axis 1-1 or axis 2-2).



Solution 11.3-11 Column with restraint at midheight

W 10 × 45 $E = 30 \times 10^6$ psi

$L = 18$ ft $= 216$ in. $I_1 = 248$ in.^{4} $I_2 = 53.4$ in.^{4}

$n = 2.4$

BUCKLING ABOUT AXIS 1-1

$$P_{cr} = \frac{\pi^2 EI_1}{(2L)^2} = 393.5\,\text{k}$$

BUCKLING ABOUT AXIS 2-2

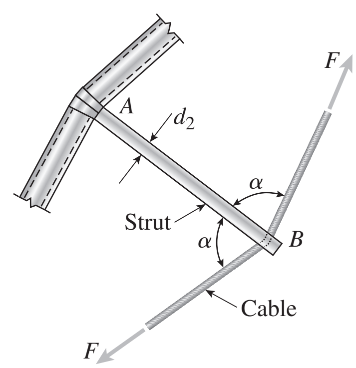
$$P_{cr} = \frac{\pi^2 EI_2}{L^2} = 338.9\,\text{k}$$

ALLOWABLE LOAD

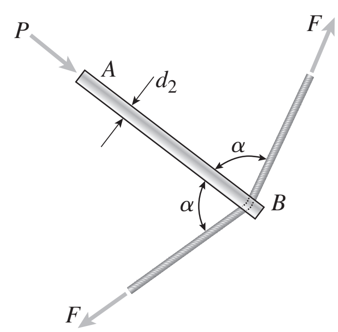
$$P_{allow} = \frac{P_{cr}}{n} = \frac{338.9\,\text{k}}{2.4} = 141\,\text{k} \longleftarrow$$

Problem 11.3-12 The multifaceted glass roof over the lobby of a museum building is supported by the use of pretensioned cables. At a typical joint in the roof structure, a strut *AB* is compressed by the action of tensile forces *F* in a cable that makes an angle $\alpha = 75°$ with the strut (see figure). The strut is a circular tube of aluminum ($E = 72$ GPa) with outer diameter $d_2 = 50$ mm and inner diameter $d_1 = 40$ mm. The strut is 1.0 m long and is assumed to be pin-connected at both ends.

Using a factor of safety $n = 2.5$ with respect to the critical load, determine the allowable force *F* in the cable.



Solution 11.3-12 Strut and cable



P = compressive force in strut

F = tensile force in cable

α = angle between strut and cable $= 75°$

PROPERTIES OF STRUT $E = 72$ GPa

$d_2 = 50$ mm $d_1 = 40$ mm $L = 1.0$ m

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 181.13 \times 10^3\,\text{mm}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = 128.71\,\text{kN}$$

$$P_{allow} = \frac{P_{cr}}{n} = \frac{128.71\,\text{kN}}{2.5} = 51.49\,\text{kN}$$

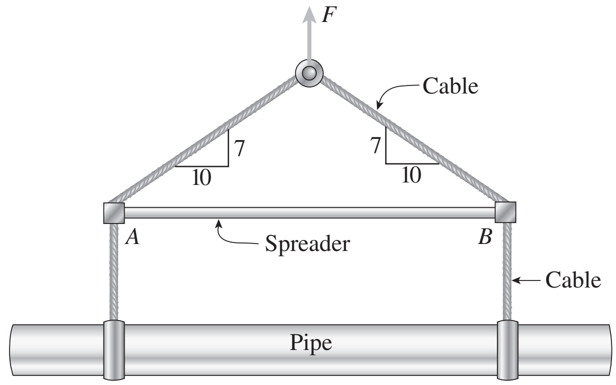
EQUILIBRIUM OF JOINT *B*

$P = 2F\cos 75°$

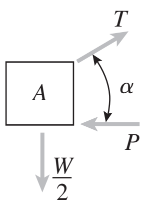
$$\therefore\ F_{allow} = \frac{P_{allow}}{2\cos 75°} = 99.5\,\text{kN} \longleftarrow$$

Problem 11.3-13 The hoisting arrangement for lifting a large pipe is shown in the figure. The spreader is a steel tubular section with outer diameter 2.75 in. and inner diameter 2.25 in. Its length is 8.5 ft and its modulus of elasticity is 29×10^6 psi.

Based upon a factor of safety of 2.25 with respect to Euler buckling of the spreader, what is the maximum weight of pipe that can be lifted? (Assume pinned conditions at the ends of the spreader.)



Solution 11.3-13 **Hoisting arrangement for a pipe**



T = tensile force in cable
P = compressive force in spreader
W = weight of pipe

$$\tan\alpha = \frac{7}{10}$$

Properties of spreader $E = 29 \times 10^6$ psi

$d_2 = 2.75$ in. $d_1 = 2.25$ in. $L = 8.5$ ft $= 102$ in.

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 1.549\ \text{in.}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = 42.61\ \text{k}$$

$$P_{allow} = \frac{P_{cr}}{n} = \frac{42.61\ \text{k}}{2.25} = 18.94\ \text{k}$$

Equilibrium of joint A

$$\sum F_{horiz} = 0 \qquad -P + T\cos\alpha = 0$$

$$\sum F_{vert} = 0 \qquad T\sin\alpha = \frac{W}{2} = 0$$

Solve the equation

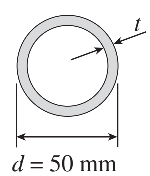
$$W = 2P\tan\alpha$$

Maximum weight of pipe

$$W_{max} = 2P_{allow}\tan\alpha = 2(18.94\ \text{k})(0.7)$$
$$= 26.5\ \text{k} \longleftarrow$$

Problem 11.3-14 A pinned-end strut of aluminum ($E = 72$ GPa) with length $L = 1.8$ m is constructed of circular tubing with outside diameter $d = 50$ mm (see figure). The strut must resist an axial load $P = 18$ kN with a factor of safety $n = 2.0$ with respect to the critical load.

Determine the required thickness t of the tube.



Solution 11.3-14 **Aluminum strut**

$E = 72$ GPa $\quad L = 1.8$ m

Outer diameter $d = 50$ mm

t = thickness

Inner diameter $= d - 2t$

$P = 18$ kN $\quad n = 2.0$

CRITICAL LOAD $\quad P_{cr} = nP = (2.0)(18 \text{ kN}) = 36$ kN

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad \therefore I = \frac{P_{cr}L^2}{\pi^2 E} = 164.14 \times 10^3 \text{ mm}^4$$

MOMENT OF INERTIA

$$I = \frac{\pi}{64}[d^4 - (d - 2t)^4] = 164.14 \times 10^3 \text{ mm}^4$$

REQUIRED THICKNESS

$d^4 - (d - 2t)^4 = 3.3438 \times 10^6$ mm^4

$(d - 2t)^4 = (50 \text{ mm})^4 - 3.3438 \times 10^6$ mm^4

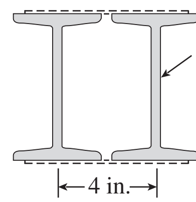
$= 2.9062 \times 10^6$ mm^4

$d - 2t = 41.289$ mm

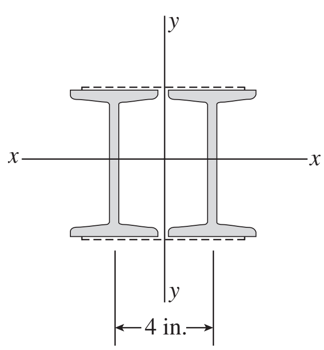
$2t = 50$ mm $- 41.289$ mm $= 8.711$ mm

$t_{min} = 4.36$ mm ⟵

Problem 11.3-15 The cross section of a column built up of two steel I-beams (S 6 × 17.25 sections) is shown in the figure on the next page. The beams are connected by spacer bars, or *lacing*, to ensure that they act together as a single column. (The lacing is represented by dashed lines in the figure.) The column is assumed to have pinned ends and may buckle in any direction. Assuming $E = 30 \times 10^6$ psi and $L = 27.5$ ft, calculate the critical load P_{cr} for the column.



Solution 11.3-15 **Column of two steel beams**



S 6 × 17.25

$E = 30 \times 10^6$ psi

$L = 27.5$ ft $= 330$ in.

$I_1 = 26.3$ in.^{4}

$I_2 = 2.31$ in.^{4}

$A = 5.07$ in.^{2}

COMPOSITE COLUMN $\quad I_x = 2I_1 = 52.6$ in.^{4}

$$I_y = 2(I_2 + Ad^2) \qquad d = \frac{4 \text{ in.}}{2} = 2 \text{ in.}$$

$I_y = 2[2.31 \text{ in.}^4 + (5.07 \text{ in.}^2)(2 \text{ in.})^2]$

$= 45.18$ in.^{4} $\quad I_y < I_x$

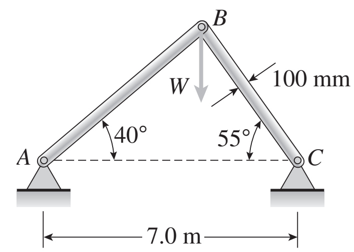
$\therefore$ Buckling occurs about the y axis.

CRITICAL LOAD

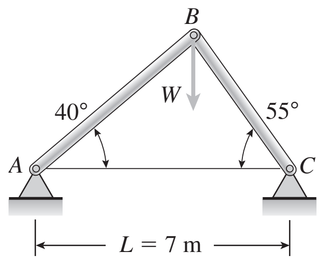
$$P_{cr} = \frac{\pi^2 EI_y}{L^2} = 123 \text{ k}$$ ⟵

Problem 11.3-16 The truss *ABC* shown in the figure supports a vertical load *W* at joint *B*. Each member is a slender circular steel pipe ($E = 200$ GPa) with outside diameter 100 mm and wall thickness 6.0 mm. The distance between supports is 7.0 m. Joint *B* is restrained against displacement perpendicular to the plane of the truss.

Determine the critical value W_{cr} of the load.



Solution 11.3-16 **Truss *ABC* with load *W***



STEEL PIPES *AB* AND *BC*

$E = 200$ GPa $\quad L = 7.0$ m

$d_2 = 100$ mm $\quad t = 6.0$ mm

$d_1 = d_2 - 2t = 88$ mm

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 1.965 \times 10^6 \text{ mm}^4$$

LENGTHS OF MEMBERS *AB* AND *BC*

Use the law of sines (see Appendix C)

$$L_{AB} = L\left(\frac{\sin 55°}{\sin 85°}\right) = 5.756 \text{ m}$$

$$L_{BC} = L\left(\frac{\sin 40°}{\sin 85°}\right) = 4.517 \text{ m}$$

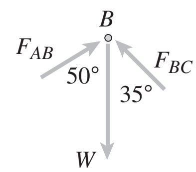
Buckling occurs when either member reaches its critical load.

CRITICAL LOADS

$$(P_{cr})_{AB} = \frac{\pi^2 EI}{L_{AB}^2} = 117.1 \text{ kN}$$

$$(P_{cr})_{BC} = \frac{\pi^2 EI}{L_{BC}^2} = 190.1 \text{ kN}$$

FREE-BODY DIAGRAM OF JOINT *B*



$$\sum F_{horiz} = 0 \qquad F_{AB} \sin 50° - F_{BC} \sin 35° = 0$$

$$\sum F_{vert} = 0 \qquad F_{AB} \cos 50° + F_{BC} \cos 35° - W = 0$$

SOLVE THE TWO EQUATIONS

$W = 1.7368\, F_{AB} \qquad W = 1.3004\, F_{BC}$

CRITICAL VALUE OF THE LOAD *W*

Based on member *AB*: $W_{cr} = 1.7368\,(P_{cr})_{AB} = 203$ kN

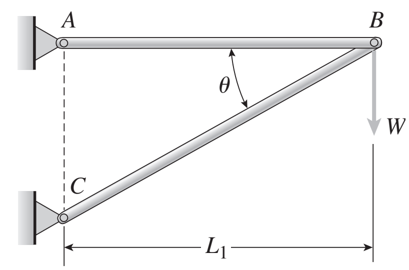
Based on member *BC*: $W_{cr} = 1.3004\,(P_{cr})_{BC} = 247$ kN

Lower load governs. Member *AB* buckles.

$W_{cr} = 203$ kN ←

Problem 11.3-17 A truss ABC supports a load W at joint B, as shown in the figure. The length L_1 of member AB is fixed, but the length of strut BC varies as the angle θ is changed. Strut BC has a solid circular cross section. Joint B is restrained against displacement perpendicular to the plane of the truss.

Assuming that collapse occurs by Euler buckling of the strut, determine the angle θ for minimum weight of the strut.



Solution 11.3-17 Truss ABC (minimum weight)

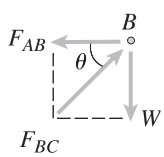
LENGTHS OF MEMBERS

$L_{AB} = L_1$ (a constant)

$$L_{BC} = \frac{L_1}{\cos\theta} \text{(angle } \theta \text{ is variable)}$$

Strut BC may buckle.

FREE-BODY DIAGRAM OF JOINT B



$\sum F_{\text{vert}} = 0 \qquad F_{BC}\sin\theta - W = 0$

$$F_{BC} = \frac{W}{\sin\theta}$$

STRUT BC (SOLID CIRCULAR BAR)

$$A = \frac{\pi d^2}{4} \qquad I = \frac{\pi d^4}{64} \qquad \therefore\ I = \frac{A^2}{4\pi}$$

$$P_{\text{cr}} = \frac{\pi^2 EI}{L_{BC}^2} = \frac{\pi E A^2 \cos^2\theta}{4L_1^2}$$

$$F_{BC} = P_{\text{cr}} \quad \text{or} \quad \frac{W}{\sin\theta} = \frac{\pi E A^2 \cos^2\theta}{4L_1^2}$$

Solve for area A: $\quad A = \dfrac{2L_1}{\cos\theta}\left(\dfrac{W}{\pi E\sin\theta}\right)^{1/2}$

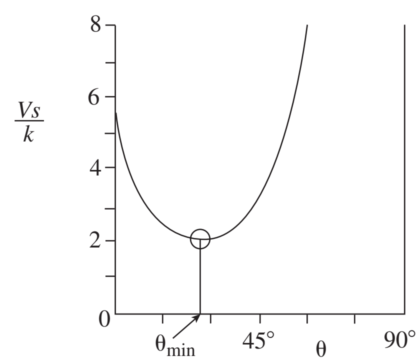
For minimum weight, the volume V_S of the strut must be a minimum.

$$V_S = AL_{BC} = \frac{AL_1}{\cos\theta} = \frac{2L_1^2}{\cos^2\theta}\left(\frac{W}{\pi E\sin\theta}\right)^{1/2}$$

All terms are constants except $\cos\theta$ and $\sin\theta$. Therefore, we can write V_S in the following form:

$$V_S = \frac{k}{\cos^2\theta\sqrt{\sin\theta}} \quad \text{where } k \text{ is a constant.}$$

GRAPH OF $\dfrac{V_S}{k}$



θ_{min} = angle for minimum volume (and minimum weight)

For minimum weight, the term $\cos^2\theta\sqrt{\sin\theta}$ must be a maximum.

For a maximum value, the derivative with respect to θ equals zero.

Therefore, $\dfrac{d}{d\theta}(\cos^2\theta\sqrt{\sin\theta}) = 0$

Taking the derivative and simplifying, we get
$\cos^2\theta - 4\sin^2\theta = 0$

or $1 - 4\tan^2\theta = 0$ and $\tan\theta = \dfrac{1}{2}$

$$\therefore\ \theta_{\text{min}} = \arctan\frac{1}{2} = 26.57° \quad \longleftarrow$$